Math 10A with Professor Stankova
Quiz 4; Wednesday, 9/20/2017
Section #107; Time: 11 AM
GSI name: Roy Zhao **Name:** _____

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True **FALSE** If a function f has a local minimum at $x = c$, then $f''(c) > 0$.

2. True **FALSE** For a function $f : [a, b] \to \mathbb{R}$, the set of critical points of f is $\{x \in [a, b] : f'(x) = 0\}$.

Show your work and justify your answers. Please include all units in the final answer.

3. (10 points) Susie is filling up an upside down conical container with paint (so the point faces down). The container measures with a total height of $2m$ and total width of $2m$.

   (a) (6 points) Write a formula that expresses the total volume V of paint in container as a function of the height h of the smaller cone that the paint forms. (Write a formula involving only V, h, and constants).

   Solution: The formula for the volume of a cone is $\frac{\pi}{3}r^2h$ and so we need to find r in terms of h. By similar triangles, we know that $\frac{h}{r} = \frac{2}{1} = 2$ where the last fraction comes from the fact that the height of the cone container is 2 and the radius of the container is $2/2 = 1m$. So $h = 2r$ and $r = \frac{h}{2}$. Thus we get the formula:
   $$V = \frac{\pi}{12}h^3.$$

   (b) (2 points) Assuming that Susie is filling up the cone at a rate of $10^{-3}\pi m^3/s$, how fast is the height of the paint increasing when it is at a height of $1m$?

   Solution: Taking the derivative of our formula from above with respect to t gives
   $$\frac{dV}{dt} = \frac{\pi h^2}{4}\frac{dh}{dt}.$$
   Now we plug in $10^{-3}\pi$ for V' and 1 for h to get
   $$\frac{dh}{dt} = \frac{4 \cdot 10^{-3}\pi}{\pi} = 4 \cdot 10^{-3} = 4mm/s.$$

(c) (2 points) Assuming that Susie is still filling up the cone at a rate of $10^{-3}\pi m^3/s$, how fast is the height of the paint increasing when she has already poured in $\pi/12m^3$ of paint?

Solution: We use the formula that $V = \frac{\pi}{12}h^3$ and when she has already poured in $\pi/12m^3$ of paint, the height of the paint is $\pi/12 = \pi/12h^3 \implies h = 1$, so the height of the paint is $1m$. So, the rate of change of the height is the same: $4 \cdot 10^{-3} = 4mm/s$.